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Irrelevance of the Pauli principle in distant correlations between identical fermions

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Abstract. Distant correlation experiments with identical fermions (e.g. protons, as in the work of Lamehi-Rachti and Mittig) require a theory which includes the Pauli principle. This fact makes an immediate application of the usual quantum mechanical distant correlation theories (including our previous work) impossible, because the exclusion principle precludes the interpretation of particles as subsystems.

Utilising the fact that the measuring apparatuses (for one-particle measurements) are in two non-overlapping spatial regions, it is shown that there exists a mapping preserving all the relevant physical information that allows a transformation to a distinct-particle picture, in which the particles can be naturally treated as subsystems, and the relevant observables have the usual simple form. It is argued that, due to the existence of this mapping, the Pauli non-local correlations do not contribute to distant correlations between identical fermions. A negentropy measure of distant correlations is introduced and discussed. It is demonstrated that they are necessarily of dynamical origin.

1. Introduction

Among the experiments performed (Clauser and Shimony 1978, Selleri and Tarozzi 1981) to resolve the contradiction between quantum mechanics and the model of local hidden variables one has made use of identical fermions (Lamehi-Rachti and Mittig 1976). Confirming quantum mechanics it actually verified the existence of non-local quantum correlations between protons. Since the Pauli principle, which is necessarily valid for this case, is itself a source of non-local quantum correlations, one wonders (Herbut and Vujičić 1985) if it is possible to separate these Pauli identical-fermion correlations from the rest of the non-local correlations.

It is the aim of this article to show that this is indeed possible in the case of two non-overlapping regions of observation, and that, as far as distant correlations are concerned, the Pauli identical-fermion correlations do not contribute at all, and that distant correlations are entirely of dynamical origin.

2. The preparation conditions

In all distant correlation experiments one has two measuring apparatuses A and A' in non-overlapping spatial domains V and V' respectively. First we consider the case of a *two identical distant fermion* state vector Ψ_{12}^{F} describing a pure ensemble prepared so that one of the fermions, no matter which, certainly arrives at V, while the other certainly arrives at V'.

To express this preparation condition in quantum mechanical terms, we introduce two kinds of single-particle projectors:

$$P_i \stackrel{\text{def}}{=} \int_V |\boldsymbol{r}_i\rangle \, \mathrm{d}V \langle \boldsymbol{r}_i| \qquad \qquad (1a)$$

$$i = 1, 2.$$

$$P'_i \stackrel{\text{def}}{=} \int_{V'} |\boldsymbol{r}_i\rangle \, \mathrm{d}V \langle \boldsymbol{r}_i| \qquad \qquad (1b)$$

It is understood that these projectors act trivially in the spin factor space of the corresponding particle.

Owing to non-overlapping: $V \cap V' = \varnothing$, one has the important orthogonality relation

$$P_i P'_i = 0 \qquad i = 1, 2 \qquad \qquad (1c)$$

and the above preparation means that

$$(P_1 \otimes P'_2 + P'_1 \otimes P_2)\Psi^F_{12} = \Psi^F_{12}. \qquad \qquad (2)$$

In this expression $P_1 \otimes P'_2$ stands for the arrival of the first fermion at V, and simultaneously the arrival of the second one at V'. The exchange term $P'_1 \otimes P_2$ represents the situation in which the particles are exchanged, and the plus sign between the two orthogonal two-particle projectors means 'or'. The invariance of Ψ^F_{12} under the projector

$$P_1 \otimes P'_2 + P'_1 \otimes P_2$$

i.e. *the preparation condition* (2), expresses the fact that the corresponding proposition is true for the ensemble described by Ψ^F_{12}, and that the elements of the ensemble are thus adapted to the localisation and separation of the measuring apparatuses A and A'.

If we had *two distinct particles* instead of identical ones, so that particle 1 arrived at the apparatus A and particle 2 at A', the preparation condition for a two-distant-particle state vector Φ_{12} would be

$$(P_1 \otimes P'_2)\Phi_{12} = \Phi_{12} \qquad \Phi_{12} \in H_1 \otimes H_2. \qquad \qquad (3)$$

It should be noted that in the usual (distinct-particle) distant-correlation experiments the preparation condition (3) is satisfied, which means that the two-distinct-particle state space $H_1 \otimes H_2$ is restricted to

$$R_d \stackrel{\text{def}}{=} R(P_1 \otimes P'_2) = R(P_1) \otimes R(P'_2).$$

We call the description of distant correlations in this subspace *the distinct-particle picture.*

3. Transition to the distinct-particle picture

In the case of identical fermions, the non-overlapping of the spatial regions V and V' (typical for distant correlation measurements) enables one to introduce an *a posteriori distinction* of the particles: the particle that arrives at V is by definition particle 1, the one that arrives at V' is particle 2. This means replacing Ψ^F_{12}, which satisfies (2) and

$$E_{12}\Psi^F_{12} = -\Psi^F_{12} \qquad \qquad (4)$$

by a physically equivalent Φ_{12} satisfying (3). The operator E_{12} in (4) is the exchange operator in $H_1 \otimes H_2$. It exchanges the spatial and the spin coordinates.

It turns out (theorem 1) that this replacement can be achieved essentially by projecting out the $(P_1 \otimes P'_2)$ component of Ψ^F_{12}. This map preserves all the relevant physical information contained in Ψ^F_{12}, because it acts as an isomorphism (though projectors are singular operators). This will be seen to stem from the fact that we are confined to Ψ^F_{12} satisfying the preparation condition (2).

Theorem 1. The operator $2^{1/2}(P_1 \otimes P_2')$ maps the subspace R_i consisting of all Ψ_{12}^F (satisfying simultaneously the compatible conditions (2) and (4)) as an *isomorphism* onto

$$R_d \stackrel{\text{def}}{=} R(P_1 \otimes P_2').$$

The inverse isomorphism is $2^{1/2}A_{12}$, where $A_{12} \stackrel{\text{def}}{=} 2^{-1}(1 - E_{12})$ is the projector onto the antisymmetric subspace.

Proof. Since the projectors A_{12} and $(P_1 \otimes P_2' + P_1' \otimes P_2)$ commute, the set of all Ψ_{12}^F satisfying (2) and (4) is the subspace

$$R_i \stackrel{\text{def}}{=} R(A_{12}) \cap R(P_1 \otimes P_2' + P_1' \otimes P_2).$$

Further, owing to $R_d \subset R(P_1 \otimes P_2' + P_1' \otimes P_2)$, which is equivalent to

$$(P_1 \otimes P_2')(P_1 \otimes P_2' + P_1' \otimes P_2) = P_1 \otimes P_2'$$

the projector A_{12} by itself projects R_d into R_i because it leaves $R(P_1 \otimes P_2' + P_1' \otimes P_2)$ invariant. Obviously, $P_1 \otimes P_2'$ projects R_i into R_d.

Next we prove that $2^{1/2}A_{12}$ and $2^{1/2}(P_1 \otimes P_2')$ invert each other if their domains are confined to R_d and R_i respectively:

$$(2^{1/2}A_{12})(2^{1/2}P_1 \otimes P_2')[A_{12}(P_1 \otimes P_2' + P_1' \otimes P_2)]$$
$$= A_{12}(P_1 \otimes P_2' + P_1' \otimes P_2)$$
$$(2^{1/2}P_1 \otimes P_2')(2^{1/2}A_{12})(P_1 \otimes P_2') = P_1 \otimes P_2'.$$

These equalities can be easily checked using the following relations:

$$(P_1 \otimes P_2')(P_1' \otimes P_2) = 0$$
$$E_{12}(P_1 \otimes P_2') = (P_1' \otimes P_2)E_{12}$$
$$-E_{12}A_{12} = A_{12}.$$

Thus $2^{1/2}A_{12}$ and $2^{1/2}(P_1 \otimes P_2')$ are mutually inverse bijections of R_d and R_i onto each other.

To prove that they are isometric maps, it is sufficient to show this for one of them, say for $2^{1/2}A_{12}$. It is further sufficient to take $\varphi \in R(P_1)$, $\|\varphi\| = 1$, $\chi \in R(P_2')$, $\|\chi\| = 1$, and to prove that $2^{1/2}A_{12}(\varphi \otimes \chi)$ is still normed. Indeed this is so as is well known from the manner of obtaining Slater determinants.

Remark. It may be worthwhile to give a simple illustration of the basic geometrical idea in the above proof. Let us take the three-dimensional real Euclidean space E_3 as the counterpart of $H_1 \otimes H_2$. Let $R(P_1 \otimes P_2' + P_1' \otimes P_2)$ correspond to the x, y-plane, and $R(A_{12})$ to the x, z-plane. Then R_i is the analogue of the x-axis. As to R_d, let us take for its correspondent an x'-axis obtained by rotating the x-axis for 45° around the z-axis.

This rotation is the counterpart of $2^{1/2}(P_1 \otimes P_2')$ as far as its application to the x-axis goes, because it can be replaced there by the action of the projector onto the x'-axis and by subsequent multiplication with $(\cos 45°)^{-1} = 2^{1/2}$ to make up for the shortening of vectors. In this way isometry is reproduced by a renormalised projector.

The analogue of $2^{1/2}A_{12}$ is the inverse rotation which can be achieved by projection into the x, z-plane with subsequent multiplication with $(\cos 45°)^{-1} = 2^{1/2}$. This is so because in this case projection onto the x, z-plane amounts to the same as projection onto the x-axis. The reason for this is the fact that the x'-axis lies in the x, y-plane, and the latter is invariant under the projector onto the x, z-plane. Thus, here isometry is achieved by renormalised projection onto a larger space, the projector of which is in theorem 1 the elementary antisymmetriser.

The significance of theorem 1 is twofold. First, it gives a transition from the standard identical-fermion picture to a distinct-particle picture on account of the non-overlapping of the spatial regions of the apparatuses A and A' (*a posteriori* distinction of the particles). Second, as a consequence of this transition, the intricate description of the measurements on the individual particles in the pairs (one-apparatus measurements) should simplify. The next section deals with this point.

4. One-apparatus observables in the transition to a distinct-particle picture

To derive the operators representing the one-apparatus observables in the identical-particle picture, one must have two single-particle Hermitian operators O_i and O'_i for each particle and they have to be compatible with the projectors associated with the localisation in V and V':

$$[O_i, P_i] = 0 \qquad [O'_i, P'_i] = 0 \qquad i = 1, 2$$

(cf (1a, b)). We assume that these operators have purely discrete spectra (the general case of measurable observables, see von Neumann 1955, p 220), and we write them in spectral form

$$O_i = \sum_p a_p P_i^{(p)} \qquad O'_i = \sum_q b_q Q_i^{(q)} \qquad i = 1, 2. \tag{5}$$

The above commutativity with the operators implies commutativity with their eigenprojectors. Using an argument analogous to the one that led to the preparation condition (2), and taking into account the mentioned compatibility of P_i and $P_i^{(p)}$, one derives the explicit form of the observable measured by the apparatus A:

$$O_{12} = \sum_p a_p (P_1 P_1^{(p)} \otimes P'_2 + P'_1 \otimes P_2 P_2^{(p)}).$$

In this expression $P_1 P_1^{(p)} \otimes P'_2$ stands for the triple coincidence: particle 1 arrives at apparatus A, the 'pointer' shows the result a_p ($P_1^{(p)}$ has occurred), and particle 2 arrives at V'. In the exchange term $P'_1 \otimes P_2 P_2^{(p)}$ the roles of particles 1 and 2 are exchanged. In terms of the observables O_i, $i = 1, 2$ (cf (5)), one obtains:

$$O_{12} = O_1 P_1 \otimes P'_2 + P'_1 \otimes O_2 P_2. \tag{6a}$$

Analogously, the observable measured by the apparatus A' is:

$$O'_{12} = \sum_q b_q (P'_1 Q_1^{(q)} \otimes P_2 + P_1 \otimes P'_2 Q_2^{(q)})$$

$$= P'_1 O'_1 \otimes P_2 + P_1 \otimes P'_2 O'_2. \tag{6b}$$

Note that O_{12} and O'_{12} are compatible with the projector in the preparation condition (2).

From the physical point of view, the restriction to the one-apparatus observables O_{12}, O'_{12}, compatible with the preparation projector, means no loss of generality because it is the purpose of the preparation to single out this kind of measurements.

It is also important to notice that O_{12} and O'_{12} are two-particle symmetrical operators. This gives an intricate description of one-apparatus observables, but there is no simpler way because one-particle operators have no physical meaning for a two-identical-fermion system.

Theorem 2. In the transition to the distinct-particle picture, i.e. to R_d, the one-apparatus observables O_{12} and O'_{12} become the one-particle observables O_1 and O'_2 respectively (cf (6*a*, *b*) and (5)).

Proof. Owing to (6*a*), (1*c*) and theorem 1, the transform of O_{12} is

$$(2^{1/2}P_1 \otimes P'_2)O_{12}[2^{-1/2}(1-E_{12})] = (P_1 \otimes P'_2)(O_1P_1 \otimes P'_2)(1-E_{12})$$
$$= (O_1P_1 \otimes P'_2)[(P_1 \otimes P'_2)(1-E_{12})] = O_1P_1 \otimes P'_2 = O_1$$

(note that $P_1 \otimes P'_2$ is the identity operator in the distinct-particle picture). The claim regarding the transform of O'_{12} follows analogously.

The result of theorem 2 is an important simplicity of the distinct-particle picture. It allows us to state that the transformation to the distinct-particle picture reduces the identical-fermion case to the usual case of distant correlations.

The physical equivalence of the identical-particle and the distinct-particle pictures consists further, as can be easily shown, in the equality of the probability of the coincidence of the results a_p of O_{12} and b_q of O'_{12} in Ψ^F_{12} with the probability of the coincidence of a_p of O_1 and of b_q of O'_2 in the corresponding Φ_{12}. Hence, the distant correlations, which consist of these coincidence probabilities, are preserved in the transition $\Psi^F_{12} \to \Phi_{12}$. In other words, *the Pauli principle*, which is lost in this transition, *gives no contribution to the distant correlations.*

5. A measure and dynamical origin of distant correlations between identical fermions

Since we have an isomorphism between the identical-fermion and the distinct-particle pictures, we start with the latter, which is better understood.

The best adapted form of a given two-particle state vector Φ_{12} for investigation of the quantum correlations inherent in it is its Schmidt canonical form (Schrödinger 1935, Herbut and Vujičić 1976)

$$\Phi_{12} = \sum_i r_i^{1/2} \varphi_i \otimes \chi_i \tag{7a}$$

in which by definition r_i are the positive eigenvalues, and $\{\varphi_i: i=1,2,\ldots\}$ are corresponding orthonormal eigenvectors of the reduced statistical operator

$$\rho_1 \overset{\text{def}}{=} \mathrm{Tr}_2|\Phi_{12}\rangle\langle\Phi_{12}| = \sum_i r_i|\varphi_i\rangle\langle\varphi_i|. \tag{7b}$$

The vectors χ_i are evaluated by means of the partial scalar product (Herbut and Vujičić 1976)

$$|\chi_i\rangle = r_i^{-1/2}\langle\varphi_i|\Phi_{12}\rangle\rangle \qquad i=1,2,\ldots. \tag{7c}$$

They turn out orthonormal.

In order to obtain a measure of the quantum correlations contained in Φ_{12}, we suggest making use of the concept of negentropy (Lindblad 1973):

$$N(\Phi_{12}) \overset{\text{def}}{=} S(\Phi_{12}) - S(\rho_1) - S(\rho_2) \tag{8}$$

where for every statistical operator ρ, $S(\rho) \overset{\text{def}}{=} -\mathrm{Tr}\, \rho \ln \rho$, and

$$S(\Phi_{12}) \overset{\text{def}}{=} S(|\Phi_{12}\rangle\langle\Phi_{12}|) = 0$$

is the entropy (von Neumann 1955) of the pure-state statistical operator. Besides, one has

$$S(\rho_1) = S(\rho_2) = -\sum_i r_i \ln r_i$$

because $\rho_2 = \Sigma_i\, r_i |\chi_i\rangle\langle\chi_i|$ (Herbut and Vujičić 1976). Hence

$$N(\Phi_{12}) = 2 \sum_i r_i \ln r_i. \tag{9}$$

The suitability of $|N(\Phi_{12})|$ as a measure of correlations is based on the following properties.

(i) When there are no correlations, i.e. when the Schmidt canonical form has only one term $\Phi_{12} = \varphi \otimes \chi$ (ρ_1 and ρ_2 are projectors), and only in this case, $N(\Phi_{12}) = 0$.

(ii) As seen from (9), $|N(\Phi_{12})|$ is larger the more approximately equal terms the RHS of (9) contains. (This is a well known property of the entropy $S(\rho)$.) This means that the Schmidt canonical form (7*a*) has more equally important terms, i.e. a larger number of states participates equally in the correlations.

(iii) Since

$$S(\Phi_{12}) \leqslant S(\rho_1) + S(\rho_2)$$

(and this is true for any quantum state ρ_{12}), one can convert this inequality into an equality (8) with the added negentropy term. This expresses the fact that there exists a *necessary balance* between the entropies of the parts ($S(\rho_1) + S(\rho_2)$) and the negentropy of the correlations between them resulting in the zero entropy of the whole (if ρ_{12} is a pure state).

Now we are prepared to go over to the identical-fermion picture:

$$\Psi_{12}^F = 2^{1/2} A_{12} \Phi_{12} = \sum_i r_i^{1/2} [2^{-1/2} (\varphi_i \otimes \chi_i - \chi_i \otimes \varphi_i)] \tag{10}$$

where we have used theorem 1 and (7*a*).

Theorem 3. The form (10) of Ψ_{12}^F has a few important properties.

(i) It is a coherent mixture (a linear combination with positive coefficients) of (normalised) Slater determinants that are strongly orthogonal, i.e. both φ_i and χ_i are orthogonal to φ_j and χ_j respectively when $i \neq j$.

(ii) Expansion (10) is a Schmidt canonical form of Ψ_{12}^F. Hence, $\{r_i/2\colon i = 1, 2, \ldots\}$ are the positive eigenvalues of the reduced statistical operator $\rho_1^F \overset{\text{def}}{=} \mathrm{Tr}_2 |\Psi_{12}^F\rangle\langle\Psi_{12}^F|$. Each of these eigenvalues has an even degeneracy.

(iii) The negentropy of Ψ_{12}^F has two terms:

$$N(\Psi_{12}^F) = -2 \ln 2 + 2 \sum_i r_i \ln r_i. \tag{11}$$

The first term is *a measure of the Pauli correlations* inherent in the two-identical-fermion state Ψ^F_{12}. Namely, it has the same value for every two-identical distant fermion state, even when there are no distant correlations, i.e., when $\Phi_{12} = \varphi \otimes \chi$ in the distinct-particle picture. Then (10) has only one Slater determinant term, and the second term in (11) is zero. The latter term is *a measure of the distant correlations* in Ψ^F_{12} because it is preserved in the transition from one picture to the other (cf theorem 1).

Proof. Since (10) is obtained by application of the exchange operator E_{12} (cf theorem 1), it is useful to introduce explicitly the natural isomorphism E mapping H_2 onto H_1 and giving the 'same' vector. For instance, in the coordinate-spin representation E acts on some second-particle function $\chi(\boldsymbol{r}_2, \sigma_2)$ giving a first-particle function $\bar{\chi}(\boldsymbol{r}_1, \sigma_1) \overset{\text{def}}{=} \chi(\boldsymbol{r}_2 = \boldsymbol{r}_1, \sigma_2 = \sigma_1)$, i.e. the same functional dependence on the new arguments. The equivalence of E_{12} and E consists of the following:

$$E_{12}(\varphi_i \otimes \chi_i) = (E\chi_i) \otimes (E^{-1}\varphi_i)$$

which was written as $\chi_i \otimes \varphi_i$ in (10).

(i) The expressions in the square brackets in (10) are Slater determinants if and only if $\varphi_i \perp \chi_i$ when both are in the same particle space. Actually, $\varphi_i \perp \chi_j$ for any i and j. Owing to (3) and the idempotency of P_1,

$$\begin{aligned}\rho_1 &= \mathrm{Tr}_2|\Phi_{12}\rangle\langle\Phi_{12}| = \mathrm{Tr}_2(P_1 \otimes P_2')|\Phi_{12}\rangle\langle\Phi_{12}| \\ &= P_1\, \mathrm{Tr}_2(P_1 \otimes P_2')|\Phi_{12}\rangle\langle\Phi_{12}| = P_1\rho_1\end{aligned}$$

and symmetrically

$$\rho_2 = P_2'\rho_2 .$$

As a consequence, $\varphi_i = P\varphi_i$ (in any of the particle spaces), and symmetrically $\chi_j = P'\chi_j$. The orthogonality of P and P' (cf (1c)) then implies that of φ_i and χ_j. Finally, the fact that $\varphi_i \perp \varphi_j$ and $\chi_i \perp \chi_j$, $i \neq j$, was established in (7a–c).

(ii) The expansion (7a) of Φ_{12} in an orthonormal first-particle basis $\{\varphi_i : i = 1, 2, \ldots\}$ is a Schmidt canonical form if and only if this basis is an eigenbasis of ρ_1. Schrödinger's theorem (Schrödinger 1935) claims that this requirement is valid if and only if the set $\{\chi_i : i = 1, 2, \ldots\}$ is orthonormal. We use the latter necessary and sufficient condition because it is simpler. The set of second factors in (10) is $\{\chi_i : i = 1, 2, \ldots\} \cup \{-\varphi_i : i = 1, 2, \ldots\}$, and it is orthonormal as proved under (i). Consequently, (10) is a Schmidt canonical form, the orthonormal set $\{\varphi_i : i = 1, 2, \ldots\} \cup \{\chi_i : i = 1, 2, \ldots\}$ is an eigenbasis of ρ_1^F in $R(\rho_1^F)$, and each eigenvalue $r_i/2$ of ρ_1^F obviously has an even degeneracy.

(iii) $$N(\Psi^F_{12}) = 4 \sum_i (r_i/2) \ln r_i/2$$

as follows from (9) and (10), and it gives (11).

As shown in theorem 3(iii), the negentropy of distant correlations (9) is preserved in the transition from the distinct-particle to the identical-fermion picture. What is more, the coherent mixing $\{r_i : i = 1, 2, \ldots\}$ itself is preserved (cf (7a) with (10)). To prove the dynamical origin of distant correlations, we demonstrate analogous preservation in evolution without interaction.

Theorem 4. If a two-identical-fermion system in a state Ψ_{12}^F evolves without interaction, i.e. if the evolution operator has the form $U_1 \otimes U_2$, then the coherent mixing $\{r_i: i = 1, 2, \ldots\}$ (and hence the negentropy of Ψ_{12}^F) does not change. In other words, the distant correlations come about or change only due to interaction, i.e. they have a dynamical origin.

Proof. Applying $U_1 \otimes U_2$ to (10), one obtains

$$(U_1 \otimes U_2)\Psi_{12}^F = \sum_i r_i^{1/2}[2^{-1/2}(U_1\varphi_i \otimes U_2\chi_i - U_1\chi_i \otimes U_2\varphi_i)]. \tag{12}$$

Owing to the unitarity of U_1 and U_2, (12) is still an expansion in an orthonormal first-particle basis, and also the second-particle states are orthonormal. Hence, according to Schrödinger's theorem (cf proof of theorem 3(iii)), (12) is a Schmidt canonical form in terms of Slater determinants. This implies that the coherent mixing also in $(U_1 \otimes U_2)\Psi_{12}^F$ is $\{r_i: i = 1, 2, \ldots\}$.

6. Discussion

Theorems 1 and 2 can be understood to imply that the preparation condition (2) makes it impossible for the Pauli correlations to show up in the measurements. Thus, they are physically irrelevant in this case. This makes the distinct-particle picture the natural framework for describing distant correlations of identical fermions. Further, this confirms Pauli's claim (Pauli 1973) that, in case of non-overlapping spatial regions, the exclusion principle is irrelevant.

In a previous article (Vujičić and Herbut 1984) we have shown that a necessary and sufficient condition for an Einstein-Podolsky-Rosen state is the degeneracy of at least one positive eigenvalue r_i of the reduced statistical operators. In this paper it was seen that $\rho_1^F = \mathrm{Tr}_2|\Psi_{12}^F\rangle\langle\Psi_{12}^F|$ has all its r_i with even degeneracy (Kramers degeneracy, cf theorem 3(ii)). One wonders if this means that every Ψ_{12}^F is an EPR state.

In order to clarify this point, it is important to notice that the standard theory of distant correlations (including the previously mentioned article) is *not applicable* to Ψ_{12}^F. Besides the preparation condition (2), Ψ_{12}^F also satisfies condition (4) of being antisymmetric. This precludes any physical meaning of the single particles as subsystems. In particular, for Ψ_{12}^F the one-particle operators $O_1 \otimes I_2$ and $I_1 \otimes O_2$ are not physical observables at all, whereas the theory of distant correlations is based on them (see discussion A in Vujičić and Herbut (1984)).

The isomorphic transition $\Psi_{12}^F \to \Phi_{12}$ to the distinct-particle picture recovers the physical idea of subsystems lost in confining $H_1 \otimes H_2$ to its antisymmetric subspace. Since Φ_{12} does not always have Kramers degeneracy, it is not necessarily an EPR state.

Evidently, Ψ_{12}^F is an EPR state if and only if its ρ_1^F has at least one positive eigenvalue of multiplicity not less than four.

It is known (Herbut and Vujičić 1976) that every state vector $\Phi_{12} \in H_1 \otimes H_2$ has a Schmidt canonical form (7*a*). In this paper it has been established that if Ψ_{12}^F satisfies (2) and (4), then it has also a stronger Schmidt canonical form (10). It can be shown that this result is more general than the distant-correlation case explored here, i.e. it is valid without restriction (2).

Let us point out that the entire treatment of this paper is based on the use of the orthogonality condition (1*c*). One realisation of this (required by distant correlations) is given by $(1a, b)$. There are other possibilities for the choice of P_i and P'_i, $i = 1, 2$, in connection with other physical problems. For instance, isospin is a typical and well known case of equivalence of the identical-particle picture (isospin formalism) and the distinct-particle picture (the proton–neutron description (Messiah 1962)).

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